Jožef Stefan Institute, Ljubljana, Slovenia

Deterministic Radio Propagation Modelling State-of-the-Art as of 2016

ARRS Grant No. L2-7664

JSI Technical Report

DP-12242

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Ljubljana, 18.1.2017

Contents

About the report

This document provides initial survey of the development in the field of deterministic radio propagation modelling algorithms as planned in Activity A1.1 of the research project entitled Advanced Ray-Tracing Techniques in Radio Environment Characterization and Radio Localization. The project is supported by the Slovenian Research Agency under Grant No. L2-7664. It takes place at the Department of Communication Systems in collaboration with Alanta and Xlab. The latest findings in the field of deterministic algorithms, especially in the field of radio ray-tracing techniques and their computer graphics counterparts, are reviewed.

Radio ray tracing

In global illumination domain, ray tracing is based on a physical optics approximation, where paths of light rays are followed through the scene, with processing of various sorts on surface intersections in order to reproduce reality as close as possible. Ikegami et al. [1] were among the first who showed the usefulness of ray-tracing technique for radio wave propagation prediction in 1991. A larger set of electromagnetic effects are typically dealt with at radio frequencies. For example, diffraction [2] and interference are generally not considered in the global illumination problem, although some exceptions exist [3].

Raytracing supports deterministic channel modelling, as opposed to the stochastic modelling in which radio channel is approximated by parametric functions based on extensive measurements. Well known in this category are Ikegami, Wallfisch and Hata models. Short computation time of the stochastic models is offset by large prediction errors, especially in heterogeneous environments. On the other hand, raytracing allows advanced channel characteristics evaluation, such as delay spread or direction of arrival, at the cost of higher processing efforts. It was shown that ray tracing is mathematically equivalent to modal methods, which are widely used for modelling radio propagation in tunnels, both methods yielding the same results [4].

Two computationally distinct raytracing approaches have been followed since early beginnings. The first, often seen as the brute force approach, effectively traces a large number of rays from the transmitting source in all directions into the scene. The concept of a reception sphere is usually needed to detect rays passing by the receivers [5]. The algorithms from this group refer to the principle as ray launching [6], ray shooting and bouncing (SBR) [7], pincushion method [8] or more elaborated ray tube [9] and beam tracing [10], the latter aggregating rays to reduce computational complexity and effectively converging to the second approach, known as the method of images [11].

Latest surveys

General overview of the radio propagation models can be found in COST 273 report [12].

The latest survey of wireless path loss prediction and coverage mapping methods dates to 2013 [13]. Surveyed are purely theoretical models, statistical models, deterministic ray-optical models, and measurement-directed methods. A new taxonomy is also proposed.

Limited to tunnel radio environment is the survey in [14], which includes ray-tracing based methods for a specific tunnel geometry.

Small-cell and indoor application of ray tracing is reviewed in [15]. Special attention is put on diffuse scattering, multidimensional channel characterization, MIMO capacity and on the real time aspects of ray tracing.

Shooting and bouncing rays

Shooting and bouncing rays rely on the concept of a reception sphere to detect rays passing by the receivers. Double counting is a common problem addressed by the SBR algorithms. Multiple hits by rays from the same wavefront require special treatment, which can introduce some errors in the estimate of a real signal. The problem has already been addressed in earlier works. Discrimination of received rays according to the objects they encounter on their paths is proposed in [16].

Ray tracing of diffraction phenomena is based on Uniform Theory of Diffraction (UTD). A new UTD solution is proposed for triple diffracted rays in [17]. The electric field is given in an analytical form that takes into account overlapping transition regions. In addition to UTD, the Uniform Asymptotic Theory (UAT) and Spectral Theory of Diffraction (STD) have been developed in the past. The latest overview of these theories was published in 2013 [18].

Special case of vertices at the tip of a pyramid formed by curved surfaces with curvilinear edges is analysed as UTD problem extension in [19]. UTD ray solution for vertex diffraction is proposed.

The number of ray-surface intersection tests can be reduced by a volumetric space partitioning as proposed in [20], where an adaptive multi-level extension of well-known volumetric space partitioning scheme is given.

Another attempt to reduce the number of launched rays is shooting rays in direction that better match antenna transmission pattern [21]. The method can be rewarding for outdoor site-specific radio predictions.

Shooting and bouncing rays is subdivided into three phases in order to accelerate computation in [22]: pre-processing, tracing, and post-processing. The relation between the facet pairs is exploited in the pre-processing step. Further, a heuristic for removal of double traced rays is proposed. The algorithm supports up to two consecutive diffractions.

A huge amount of storage capacity is typically needed to handle reception points at a given spatial resolution. A procedure based on the concept of ray entities both to enable continuous interpolation of ray-tracing data and to reduce the memory requirements is proposed in [23]. Another paper dealing with the interpolation problem is [24]. Multiple-input-multiple-output indoor communication is considered, with ray tracing implemented only between selected pairs of transmitting and receiving antenna points. Statistical analysis is used to estimate the surrounding areas. The authors find the method highly accurate within a range of several wavelengths from the receiving point.

Method, which can be also classified as a data interpolation, is the acceleration technique based on the decomposition of wavefronts [25]. By tracing just a few rays, the algorithm finds the solid angles around the transmitter that transport electromagnetic power to the receiver. The accuracy is then improved by iteratively increasing the tessellation frequency of the source in the power-transporting solid angles. Because power-transporting solid angles constitute only a small fraction of the total space around the source through which the rays are launched, a very efficient but approximate ray tracing is possible.

Method of images

Method of images has been known since the first attempts to predict electromagnetic wave propagation for radio communications [26]. The method is designed around the observation that rays reflected from a surface seem to originate from a fictitious transmission source, which can be found symmetrically on the other side of the surface along its perpendicular [27]. Such a fictitious source is called a source image. Multiple reflections are accounted for by considering existing images as new transmission points, which recursively leads to the image tree. Surfaces have a finite shape, limiting the reflected regions to polyhedron volumes, which helps in reducing the size of a tree. If the image tree describes only viable propagation paths, then it is referred to as a visibility tree.

Image trees are generally larger than visibility trees and require elaborate intersection tests between the ray paths and scene objects in the second signal-backtracking step. Note that a backtracking is always required to re-create signal paths, even in a strict visibility tree, at least to establish the length of a path and all the reflection coefficients affecting the radio signal power.

Image trees are not limited solely to reflected paths. Through-wall transmissions can be dealt with successfully by extending the source mirroring onto walls that are identified in the so-called transmission regions [11]. On the other hand, corner diffractions can be adequately handled by the method of images only in 2D propagation scenarios [28]. Namely, a corner acts as a source of rays, generally introducing an infinite number of new sources unless it is represented by a single point. Therefore, 3D diffraction must be accounted for by some other means.

Image tree optimization

Soon after its introduction proposals to reduce the over dimensioned image tree emerged. In the following, we restrict ourselves to the published works that support full 3D environment modelling through the entire computation and to the ones being most relevant.

The elaborate method of regions [27] constrains physically feasible paths by introducing spatial regions in the shape of convex polyhedra into the image tree construction. Computing viable reflection or transmission regions translate to the polyhedra intersection problem, the solution of which involves intensive computational geometry. Simplified version of the spatial regions treatment can be found in [29], where the geometry of objects is restricted to horizontal walls of arbitrary shape and strictly rectangular vertical walls.

Instead of bounding regions, [30] deals with a set of visible surfaces contained within such feasible regions. Surfaces are represented by polygons as seen from the source image after the surface corners have been projected to the viewing plane. In order to extract polygons describing only visible parts of a surface, each projection is processed by a sweep-line algorithm, followed by the well-known graph-theoretic polygon subtraction to account for hidden parts of a surface. The reflection visibility window is represented by yet another polygon in the computation, generally hindered by the treatment of many special cases.

Visibility tree in [31] is only partially reduced image tree because it is based on a polar-sweep of 2D space. When applied to 3D scene in the second backtracking step, paths described by the tree may or may not give rise to actual paths and further checks are still needed. Further, being a hybrid 2D/3D method, ground, floors and ceilings must be treated separately.

Further attempts to reduce image tree size while keeping the support for full 3D computation involve various pre-processing steps on the input geometry, such as dividing surfaces into tiles and using the tile centre as the ray interaction point [32], or pre-computing intermediate values needed in the highly repetitive intersection tests, such as angular relations between the scene objects [33].

The problem of inaccurate transmission regions is dealt with in [34]. Although the method of images was originally restricted to the radio environments with prevailing reflection phenomena, it is also used in indoor scenarios in which through-wall transmission make a significant contribution to the received signal power. A source image translation heuristic based on the wall depth, material and field of view is proposed that improves accuracy of strict visibility trees, which gives a better fit of predicted signal to the theoretically correct solution.

Hybrid methods

Due to the high processing requirements of ray tracing in larger environments or simply to improve prediction for complex scene details, hybrid methods have been frequently proposed. Radio wave propagation through periodic building structures for site-specific outdoor-to-indoor propagation is captured in a hybrid model based on combining ray-tracing and rigorous coupled wave analysis (RCWA) [35]. In comparison to finite-difference time-domain (FDTD) analysis good agreement was reported.

Diffuse tail of impulse response simulation is difficult to capture properly by ray tracing. A hybrid model that combines ray tracing with a propagation graph is proposed in [36]. The recursive structure of the propagation graph allows for a computationally efficient calculation of the channel transfer function. The delay power spectrum and the azimuth-delay power spectrum benefit from this hybrid approach.

Image theory and SBR are merged in a quasi 3D ray-tracing heuristic in [37]. The visibility source tree is combined with the polar sweep algorithm. All possible ray paths are found while reducing the number of intersection tests.

Binary version of the particle swarm optimization (BPS) is combined with ray tracing to optimize access point location in [38]. Power levels at a mesh of receivers are first calculated for several access point locations. Network restrictions are encoded as a fitness function and signal-to-interference ratio is kept as high as possible while ray-tracing results are fed into the BPS optimizer to derive optimal access point location.

The combination of a neural network and a 3D ray launching reduces the number of launched rays [39] with the intermediate points being predicted using neural network. Gain of 80% in terms of computational efficiency is reported.

Practically important case of thin dielectric layers over perfectly electrically conducting (PEC) bodies is considered in [40]. The resulting hybrid SBR is capable of dealing with various dielectric objects in a unified way.

Hardware acceleration

GPUs

GPUs has been studied extensively in the context of shooting and bouncing rays [41][42][43]. Recent work includes ray launching based on the NVIDIA OptiX framework [44][45][46][47], GPU-based kd-tree accelerated beam tracing [48], and others.

Game engines are constantly optimized for real time performance. Simulation of multipath channels in complex outdoor and indoor scenarios exploiting the capabilities of GPUs support for 3D rays in game engines is studied in [49] and [50].

NVIDIA OptiX ray tracing engine, which is well known in computer graphics domain, has been adapted to the radio frequencies. Polarimetric wave propagation simulator is described in [51]. Performances of high frequency (60 GHz) channel simulation in OptiX and Paray ray tracer are evaluated.

An alternative to OptiX ray tracer is Rayforce [52]. Designed for massively parallel computing architectures, it is based on a graph acceleration structure with the application interface being suitable for many physics-based simulation domains. On the other hand, purely CPU based ray tracing is also proposed recently for professional rendering environments with complex geometries and incoherent ray distributions [53].

Apart from launching radio rays on GPUs, the massively parallel architecture is exploited in pre-processing steps, such as in building acceleration data structures. CUDA implementation of building kd-tree and SBVH are proposed in [54]. Radio ray tracing uses the same methods for detecting closest hits of rays with scene objects as computer graphic. The performance of the above data structures for radio ray tracing is analysed in [55] and [56]. In the cases considered, the SBVH performed better in terms of time and space. A new spatial decomposition based data structure called Shell is proposed in [57] in order to completely avoid hierarchical search for ray traversals. The authors observe that the usual hierarchical structures are not well suited to the shared memory many core architectures. A structure is built on the boundary of each region in the decomposed space with provided table mappings of neighbouring regions. Average speed-up of 4 over the kd-trees is reported.

Usually a single ray is treated within a thread on the entire path through the scene. Different work distribution is prosed in [58] in order to avoid SIMD utilization drop due to divergent ray paths. Tracing process is separated into three steps: ray-box testing, intersection step, and closest hits selection.

Despite the increasing availability of parallel hardware, running the method of images in a parallel way has been largely avoided. The reason can be related to the fact that the imaging technique is not as parallel as the SBR technique [26], for which various hardware accelerators have been intensively studied besides GPUs, such as Cell architecture [59], SaarCOR processor [60], and FPGAs [61].

The straight-forward way to parallelize the method of images is to partition the image tree and use some form of dynamic load balancing, such as splitting the area database in regions and using work-pool concept [62]. The authors characterize their approach as being fine-grained. However, it is still rather coarse and better suited for parallel computers in general and less for the GPU-like massively parallel architectures.

The closest to the fine-grained parallelization is the rasterization of beams in [10], with beams being considered equivalent to the feasibility regions. Unfortunately, Schmitz et al. designed custom rendering pipeline that can be applied only to a 2D version of propagation prediction and cannot be extended to the third dimension.

The rasterization-based graphics clearly outperforms ray tracing in current GPUs. A precise quantitative analysis is available in [63]. Computer graphics rendering of reflections and double refractions is proposed in [64] as an efficient way to construct strict visibility trees using task parallelization capabilities of the latest GPUs. Visible surfaces are identified in a graphical framebuffer using standard stencil, z-buffering and plane clipping.

ASICs and FPGAs

Using other type of hardware accelerators, such as FPGAs and ASICs, have been proposed mainly in the context of ray tracing in computer graphic. Some publications are presented in the following because the research in the field has a potential to be extended to the radio domain.

First implementations of ray tracing in FPGA were reported at the beginning of a century [65]. Back then they outperformed desktop PC architectures by up to two orders of magnitude.

Ray-object intersection tests are frequently targeted by special hardware due to their dominant processing requirements. FPGA implementation based on a plane-sphere intersection algorithm is presented in [66]. The highly efficient prototype utilizes 90% of FPGA's DSP cores. On the other hand, ray-triangle intersection test is accelerated in [67]. Communication bottleneck with acceleration hardware is addressed by enhancements in the pipeline architecture.

ASIC design with SIMD processing and massive multithreading makes a foundation of a dynamic ray processing unit in [68]. Although the prototype unit has been realized only in FPGA, the developers estimated ASIC design performance to be 70 times faster than commodity CPU implementation.

FPGA based ray-tracing platform that interacts with open-source renderer LuxRays consisting of four modules - communication, traversal, intersection and memory - is presented in [69]. [70] puts more emphasis on reduction in system memory bandwidth and on energy efficiency of the proposed accelerator.

FastTree [71] is a dedicated hardware kd-tree implementation, for which an FPGA prototype was built and ASIC implementation evaluated.

Applications

Ray tracing is used as a tool in many research problems. Planning micro radio repeater system to enhance signal coverage for the nodes of a mobile ad-hoc network [72] can be based on ray-tracing analysis. Similarly, massive ray tracing in outdoor urban areas for extremely high frequency bands (mm wave small cells) is successfully applied to the transmitter placement problem in [73]. Large computation time is reduced by 2D abstraction. Using simplified 2D approach may be acceptable for transmitter placement problem; however, if accurate information about the radio wave propagation channel is essential, the topography of the immediate surroundings should not be overlooked. In [74] measured and simulated power delay profiles are compared for 3D models of different accuracy, including one obtained by unmanned aerial vehicle. Close-range photogrammetry provided better propagation prediction.

Concave surfaces that are found in a large passenger aircraft introduce specific problems in the radio-frequency field mapping by ray tracing. Shape of enclosures leads to ray proliferation. A quasi-analytical ray-propagation model is proposed involving uniform ray launching, an intelligent scheme for ray bouncing, and an adaptive reception algorithm [75].

Tree scattering is commonly approached by statistical methods while [76] models the problem as a 3D ray-optical approach. A tree is modelled as a cylindrical volume with the scattered fields within a volume being calculated using the Foldy-Lax multiple scattering theory.

Application of ray tracing at nanoscale dimensions has been conventionally avoided. The FDTD is the method of choice when it comes to small scale environments. The analysis in [77] reveals that the ray-tracing approach can be successfully employed at nanoscale, with increasing agreement with FDTD at higher frequencies.

User is affecting wireless communication, especially if a portable terminal is leaned against the body. A user-shadowing path loss model is proposed in [78] by a statistical analysis of the ray-tracing simulations. The authors combine the single path model based on the uniform theory of diffraction (UTD) with the multipath channel profiles from the ray tracing.

Ray tracing is also used in the real-time performance evaluation system based on the time-of-arrival measurements in [79] in order to create controllable and repeatable multipath conditions.

THz communication systems

Future indoor communications will rely heavily on significantly higher carrier frequencies than used today. The significance of ray tracing for those systems is even bigger as ray tracing precision increases with frequency. Models to approximate and reproduce the distance-dependent behaviour of the angular spread as well as of the RMS delay spread are proposed in [80]. Further, the maximum symbol rates achievable without any inter-symbol interference are quantified.

Rough surface scattering is rarely simulated in indoor predictions due to large wavelength of radio frequencies. However, in THz communication systems scattering becomes more important. Ray-tracing algorithm based on the binary space partitioning and a diffuse scattering algorithm based on the Oren-Nayar's theory is proposed to account for scattering effect in [81]. Some original solutions eliminate the repeated ray-object intersection tests.

Localization

Ray multipath channel model is getting increased attention for the radio-based localization. Experiments using a radio receiver equipped with a low-cost accelerometer and gyroscope are described while simultaneously constructing a map of the small-scale fading pattern in [82]. The sensor data is combined with the ray-based radio channel model with unknown transmitter location. The authors obtain state vector using a particle filter. Significant improvement in long-term positioning performance is reported.

Random sampling of 3D radio frequency fingerprints for localization is challenged in [83]. Fingerprint data is represented as tensors. The use of tensor algebraic methods for an adaptive tubal sampling of fingerprint space is proposed, which can lead to significant improvements in localization accuracy. Indoor ray-tracing is used for validation.

Large scale FDTD

The finite-difference time-domain method is considered to be more accurate implementation of the laws of electromagnetic propagation as any ray-tracing would ever be, essentially simulating core Maxwell equations. Since FDTD simulations require very large number of iterations, they are only effective in small scale scenarios. With the advent of hardware accelerators and with the progress in the field of massive parallelism their reach is steadily expanding and pose, at least in medium scale scenarios, real competition to the radio ray-tracing techniques. Here we survey the latest achievements in the field.

FDTD on GPUs

GPU implementations of FDTD are widespread [84]. First papers on the topic appeared shortly after the release of Compute Unified Device Architecture (CUDA) technology. With the new technology, aims were set high, such as the use of FDTD to predict radio coverage in [85]. A year later implementations spanning entire clusters emerged. Acceleware's G80 GPU cluster managed to simulate 3GCells with throughput of up 13 GCells/s. 4 and 16-node configurations achieved efficiency of 80% and 60% at 25 to 29 times CPU cluster speed of comparable size [86].

Tutorial-like introduction on the two-dimensional human-antenna interaction is presented in [87]. A comparison of FDTD implementation on various parallel computing architectures, from multi-core processors to GPUs, showed favourable benefits of using extensive parallelism [88].

Several FDTD variants have been successfully ported to GPUs. TEz-FDTD inside piecewise-linear recursive-convolution dispersive media, truncated by the convolutional perfectly matched layer is studied in [89]. 3D FDTD was applied to the large-scale numerical human model in biomedical engineering in [90]. Four NVIDIA Tesla C2070 GPGPUs is shown to be slightly slower (1.3 times) than a significantly costlier supercomputer. Alternating-direction-explicit (ADI) FDTD implementation on GPUs is proposed in [91]. The method is important in the field as it is almost unconditionally stable algorithm. Hybrid implicit-explicit FDTD for the analysis of the shielding of printed circuit board is considered for GPUs in [92], with the reported performance superior over the CPU implementation. The efficiency of fundamental ADI FDTD and locally-one-dimensional (LOD)-FDTD on GPUs are discussed in [93]. GPU parameters for both Tesla- and Fermi-based architectures are tuned for best performance in [94].

Meshless technique for a large-scale simulation of electromagnetic wave propagation is numerically investigated in [95]. Speed up of time evolution on GPU is 8.8 over CPU, with the shape function generation achieving even three orders of magnitude.

FDTD time stepping near stability limit is penalized by slow convergence. Optimal time step selection for improved precision on GPUs is examined in [96]. Hybrid implicit-explicit FDTD combined with conformal FDTD is applied to large electromagnetic simulations in [97]. Several GPUs are employed. The comparison of GPUs with MIC architecture while running FDTD with PML absorbing boundary condition favoured GPU with CUDA over MIC with OpenMP in [98]. Discrete Green's function (DGF) FDTD is compared to FDTD grid method on GPU in [99], achieving 6-times speedup over multicore CPU.

Electromagnetic wave propagation over ocean is the largest scale FDTD attempted to date [100]. Moving window FDTD is applied to reduce memory requirements, while GPUs and OpenCL are used to reduce simulation time. Note that a moving window captures EM wave with most energy content.

FDTD using FPGAs

Custom hardware for FDTD calculation based on FPGA was proposed in 2002 [101]. A pipelined bit-serial arithmetic architecture for one-dimensional FDTD is described.

The FDTD applied to buried object detection problem implemented entirely in FPGA [102] is a representative of the pseudo-2D FDTD algorithms, which can be upgraded to 3D with some modifications.

According to the authors, the first three-dimensional FDTD accelerator implemented in physical hardware dates back to 2003 [103]. A high-level view of a system architecture and basic functionality of each module is given with performance measurements that suggest potential replacement of entire PC clusters. Speedups of classical Yee algorithm from 1966 are reported in 2004 to achieve up to three orders of magnitude over single processor solutions [104].

The objective in [105] is to reduce the number of data transfers from external memory and to study calculation error if fixed-point arithmetic is applied to the FDTD. The error of less than 0.01 is reported when the bit length is greater than 28 bits. Single precision floating point implementation is studied in [106], which was published around the same time.

Dataflow machine based on the FPGA is used in the proposal of a 3D FDTD dedicated computer in [107] and [108]. The digital circuit supports eight normal grids or four perfectly matched layer grids with almost no unused hardware resources. The dedicated FDTD computer builds on the authors' earlier work on parallel memory-access architecture [109][110].

Maxeler dataflow computer is exploited in [111] in order to overcome the fact that a FDTD algorithm is memory bound. Special attention is paid to Dirichlet, periodic and absorbing boundary conditions. Increasing memory bandwidth is also priority of the FPGA architecture for the 3D FDTD based on Open CL in [112]. In their latest publication, the authors report FPGA design at 114 GFLOPS [113], corresponding to 13 and 4 times speed-up over CPU and GPU implementations, respectively.

Multi-core processor embedded in FPGA is studied with respect to power efficiency while solving FDTD in [114]. Application-Specific Instruction Set Processor (ASIP) is used in multi-core setup with proven power consumption benefits.

SIMD array processor in FPGA is applied to the FDTD in [115]. Virtual processing-elements cuboid with synchronous shift data transfer minimize data transfer overhead, achieving 3.1 speed-up over NVIDIA Tesla implementation.

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Acronyms

MI	Method of Images
SBR	Shooting and Bouncing Rays
FPGA	Field Programmable Gate Array
GPU	Graphical Processing Unit
MIMO	Multiple Input Multiple Output
RCWA	Rigorous Coupled Wave Analysis
FDTD	Finite Difference Time Domain
RMS	Root Mean Square
AP	Access Point
UTD	Uniform Theory of Diffraction
PML	Perfectly Matched Layer
CEM	Computational Electromagnetic
ASIP	Application Specific Instruction Set Processor
SIMD	Single Instruction Multiple Data
UAT	Uniform Asymptotic Theory
STD	Spectral Theory of Diffraction
PEC	Perfect Electrical Conductor
CUDA	Compute Unified Device Architecture
SBVH	Split Bounding Volume Hierarchy
LOD	Locally One Dimensional
ADI	Alternating Direction Explicit
MTDM	Meshless Time Domain Method
MIC	Many Integrated Core
DGF	Discrete Green's Function